

Nonlinear optical responses and gap-soliton lattices in layered dielectric materials.

Alain M. Dikandé

Département de Physique, Faculté des Sciences, Université de Douala BP 24157 Douala, Cameroun.

Timoléon C. Kofané

*Laboratoire de Mécanique, Département de Physique,
Faculté des sciences, Université de Yaoundé BP 812 Yaoundé, Cameroun.*

(Dated: February 8, 2008)

A new approach to the nonlinear response of a multiplayer structure consisting of alternating dielectric materials with intensity-dependent dielectric constants is considered in both analytical and numerical viewpoints. The nonlinear feedback of the structure gives rise to an artificial potential acting as a mean-field feedback on each single dielectric material due to other layers. For superlattices resulting from a regular periodic arrangement of an infinite number of single dielectric layers, it is shown that an analytical expression of this mean-field feedback can be obtained and waveshapes of the associate gap solitons are derived. The case of a multilayer with dispersive interlayer interactions is considered numerically and leads to strongly localized gap solitons. Implications of the artificial soliton structure as for possible theoretical modelling of soliton-compression phenomena are discussed.

PACS numbers: 42.55.Tv, 42.65.Sf, 42.65.Tg

Superlattices with Bragg reflectors are generally not accessible to any light frequency. In multilayer structures consisting of a periodic arrangement of single dielectric materials, the dependence of dielectric constants on the light intensity affects the intimate structure of materials leading to the formation of defects which are responsible for these apparent gap excitations. Chen and Mills [1] pointed out a mechanism so-called stop-gap radiation transmission following which the light could still propagate within the superlattice due to partial closing of the gap defect. Later, it was suggested[2, 3] that a suitable arrangement of single dielectric materials composing a superlattice structure too may favor gap-soliton propagation. In recent years, a great deal of interest has been devoted to understanding nonlinear features of photonic band-gaps [4, 5, 6] to both view points of their structural and dynamical properties. Crudely speaking, a nonlinear photonic band-gap excitation is an electromagnetic wave moving trapped to a defect in the dielectric material with an exponentially localized shape.

Our aim in this work is to suggest a new approach to the global nonlinear feedback resulting from the superposition of the optical nonlinear properties of each single dielectric layer in a multilayer structure. In general, the propagation of electromagnetic field in a single dielectric layer is governed by the nonlinear Schrödinger equation(NLSE) i.e. [1]:

$$i\phi_t + \phi_{zz} + J_o |\phi|^2 \phi = 0. \quad (1)$$

Solutions of this equation which take account of the localized feature of the light due to a local nonlinear feedback are stationary solitons. For a self-focusing nonlinear feedback, the stationary soliton is bell-shaped(pulse) and is

obtained as:

$$\begin{aligned} \phi(z, t) &= u(z) \exp(-i\omega t), \\ u(z) &= u_o \operatorname{sech}(\sqrt{\omega} z), \quad u_o^2 = \frac{2\omega}{J_o}, \end{aligned} \quad (2)$$

In the spirit of the NLSE, this solution describes an harmonic time oscillation hidden in a spatial shell with a nonlinear topological shape. If we assume ω to carry the optical information, then by virtue of its topological feature the nonlinear spatial excitation will act as a shell protector. To a mathematical viewpoint, (2) is a spatial topological soliton with an internal frequency hence a breather soliton. Referring to the NLSE and from the analytical expression of the soliton waveshape, ω will depend on the intrinsic parameters of the breather. Thus (2) is a gap soliton, where u_o is the amplitude of the soliton gap. If we now assume a superlattice resulting from periodic arrangement of single dielectric layers obeying equation (1), gap solitons of each of these single dielectric materials become their fundamental modes. However, owing to the broken translational invariance from the coupling of multiple layers, these fundamental modes will no more obey single NLSEs and the dynamics of a given n^{th} optical component is from now on governed by the following equation:

$$i\phi_{nt} + \phi_{nzz} + \sum_{m=n}^N J_{m-n} |\phi_m|^2 \phi_n = 0. \quad (3)$$

We will assume all dielectric materials to be identical and denote their common fundamental frequency by Ω_o . The key point of our study is the assumption that equation (3) is self-consistent, i.e. is valid for all optical components. Namely, the limit $N = m$ should reproduce the single-pulse soliton (2). In this context, the sum in m runs over the entire superlattice and therefore

takes into account all fundamental modes. Doing so, the problem turns to that of a single dielectric layer in a potential bath(hereafter denoted "mean-field potential") created by its surroundings. This mean-field potential is adiabatic in the sense that it is switched on after soliton defects have formed on all the N dielectric layers. To allow for the interaction among solitons, we need to separate their centres of mass from their spatial coordinates on layers. In fact, this amounts to introduce a pinning-center in the overall coordinate of each fundamental soliton so that the isolated single-pulse is readily rewritten:

$$u(z) = u_o \operatorname{sech} \sqrt{\Omega_o}(z - z_n). \quad (4)$$

With this more appropriate shape, equation (3) leads to the following equation for the spatial wavefunction of the light wave passing through a given n^{th} optical component

and interacting with an m^{th} one via a coupling J_{m-n} :

$$u_{nzz} + \left[\omega_n + \frac{\Omega_o}{J_o} \sum_{m=n}^N J_{m-n} \operatorname{sech}^2 \sqrt{\Omega_o}(z - z_m) \right] u_n = 0. \quad (5)$$

Below, nonlinear solutions of this equation are discussed assuming two different contexts. We first consider the case of a uniform periodic arrangement of dielectric layers(regularly distributed single-layer feedbacks), and next the case of a dispersive coupling(dispersion-induced localized single-layer feedbacks) between optical components.

For $J_{m-n} = J_o$ where J_o is a constant, the mean-field potential experienced by the n^{th} optical component is the sum over sech^2 functions. This sum is exact if we note the following relevant identity($\ell = m - n$):

$$\sum_{\ell=-\infty}^{\infty} \operatorname{sech}^2 \left[\sqrt{\Omega_o}(z - \ell L) \right] = \left(\frac{2}{\pi} \right)^2 E' K' - \left(\frac{\nu K'}{E'} \right)^2 \operatorname{sn}^2 \left(\frac{z}{d_\nu} \right), \quad d_\nu = \frac{\pi}{2K' \sqrt{\Omega_o}}, \quad L = 2K d_\nu, \quad 0 \leq \nu \leq 1. \quad (6)$$

where K , K' and E' are Complete Elliptic Integrals and sn is the Jacobi elliptic function of modulus ν and period L . This relation follows from simple transformations of the usual Fourier series representation of Jacobi Elliptic functions [7], taking into consideration the extra argument z_n . The above identity, given in terms of the intensities of the local fundamental light waves, mimics a periodic lattice of single pulses mutually trapped at their centre-of-mass $z_n = nL$ where L is the separation between two interacting pulses. L increases as ν is increased so that one can monitor the degree of compression of single pulses within the potential, namely by tuning the thickness d_ν of the mean-field potential wells which according to (6) is proportional to the width of the single-pulse defect($\frac{1}{\sqrt{\Omega_o}}$). On fig. 1 we sketch the space-dependent part of the global feedback potential(i.e. the sn^2 function). When $\nu = 1$ the mean-field potential reduces to $\operatorname{sech}^2 \sqrt{\Omega_o} z$. In connection with this finding, below we will also see that we recover the original single-pulse soliton. Remark that in this limit the separation L between the single pulses is infinite.

Replacing the identity (6) in equation (5) we find:

$$\left[\frac{\partial^2}{\partial x^2} + P_\nu(\omega_n) - 2\nu^2 \operatorname{sn}^2(x) \right] u_n(x) = 0, \quad P_\nu(\omega_n) = \left(\frac{\pi}{2K'} \right)^2 \frac{\omega_n}{\Omega_o} + \frac{2E'}{K'}, \quad x = \frac{z}{d_\nu}, \quad (7)$$

Equation (7), so-called first-order Lamé equation [8], is

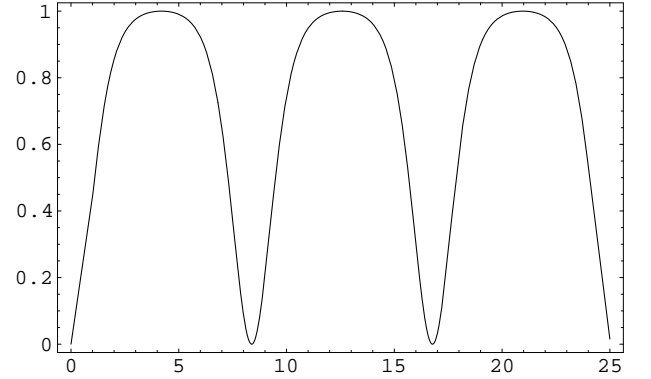


FIG. 1: The global nonlinear feedback potential.

the actual one governing the spatial waveshape of the bunch of individual solitons passing through the n^{th} optical component of the periodic lattice. This equation possesses three distinct bound states listed as [9, 10, 11];

$$u_1(x) = u_{o1}(\nu) \operatorname{dn}(x), \quad \omega_o = \Omega_o K' [k^2 K' - 2E'] \left(\frac{2}{\pi} \right)^2, \quad (8)$$

$$u_2(x) = u_{o2}(\nu) \operatorname{sn}(x), \quad \omega_1 = \Omega_o K' [(1 + k^2) K' - 2E'] \left(\frac{2}{\pi} \right)^2. \quad (9)$$

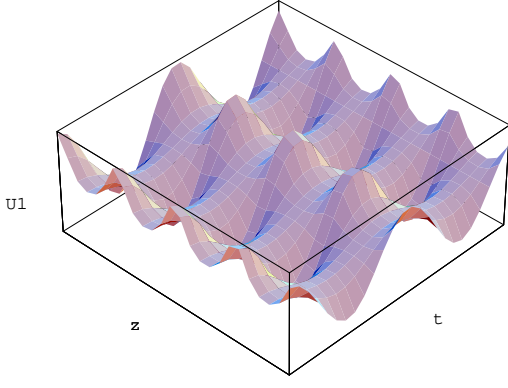


FIG. 2: Spatio-temporal wavelike shape of the dn gap-soliton lattice.

$$u_3(x) = u_{o3}(\nu) cn(x),$$

$$\omega_2 = \Omega_o K' [K' - 2E'] \left(\frac{2}{\pi} \right)^2, \quad (10)$$

In general, these bound states form a complete orthonormal set and are normalized accordingly. However, to avoid a divergent light intensity we bind their amplitudes to the amplitude of the single-soliton gap u_o . The normalization condition then becomes:

$$\int_{-L'/2}^{L'/2} u_i(x) u_j(x) dx = u_o^2 \delta_{i,j}, \quad L' = \frac{L}{d_\nu}, \quad (11)$$

and leads to:

$$u_{o1}^2 = \frac{u_o^2}{E},$$

$$u_{o2}^2 = \frac{u_o^2 \nu^2}{[K - E]},$$

$$u_{o3}^2 = \frac{u_o^2 \nu^2}{[E - (1 - \nu^2)K]}, \quad (12)$$

We can check that in the limit $\nu \rightarrow 1$ these bound states reduce to:

$$u_1(z) = u_3(z) = u_o \operatorname{sech} \sqrt{\Omega_o} z, \quad \omega_o = \omega_2 = -\Omega_o,$$

$$u_2(z) = 0, \quad \omega_1 = 0. \quad (13)$$

On figs. 2, 3 and 4 we display the spatio-temporal wavelike shapes ($\nu = 0.98$) of the three bound states.

According to their asymptotic behaviours, the first and third bound states are two degenerate periodic gap solitons. The second bound state, which gives rise to a vanishing single kink in the limit $L \rightarrow \infty$, is the state of broken translational symmetries of individual solitons due to the correlations of their (non-zero) centre-of-mass coordinates. The vanishing of this mode in the limit $\nu \rightarrow 1$ (where the separation between single pulses becomes infinite) is a clear evidence of restored translational invariance.

Let us now turn to the case of a dispersive coupling.

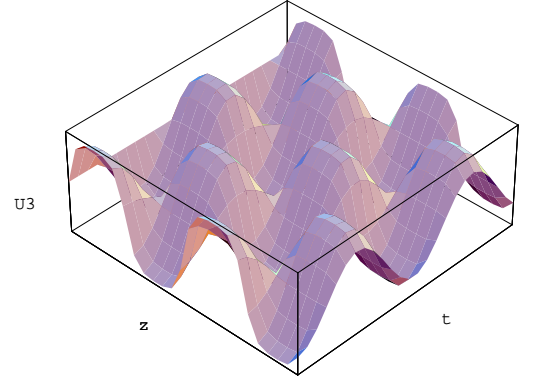


FIG. 3: Spatio-temporal wavelike shape of the sn gap-soliton lattice.

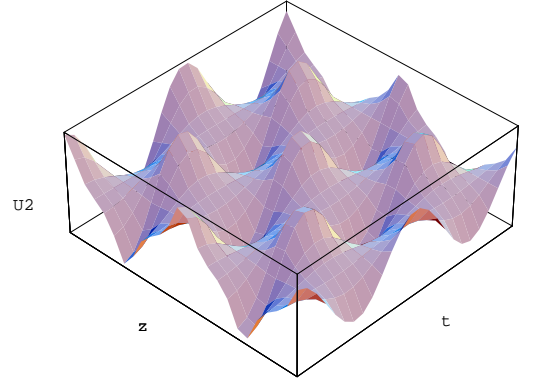


FIG. 4: Spatio-temporal wavelike shape of the cn gap-soliton lattice.

We will assume the simplest case of an exponentially decreasing interaction between single dielectric layers i.e. $J_{m-n} = J_o e^{-\lambda|m-n|}$ [12]. Since the problem cannot be efficiently treated by analytical means, we followed a numerical approach. Figs. 5, 6, 7 and 8 are shapes of the light intensity on a single layer within the multilayer structure, simulated assuming four different values of the separation L between $N = 100$ single pulses. The pulse width is 10 units corresponding to a fundamental frequency $\Omega_o \sim 0.01$, while the soliton-lattice frequency $\omega_n = 10\Omega_o$. λ is set to 0.5 which we expect to ensure relatively long distance effects. The most striking feature in these figures is a less and less periodic distribution of single pulses in the gap-soliton lattice as their separation L decreases. The gap-soliton lattice is clearly seen to transform into a bunched soliton state showing strong localization on single layers as solitons in the bunch get more and more compressed. Remark that fig. ?? corresponds to an inter soliton separation which is equal the single-pulse width. Actually, the behaviours emerging from the figures are not surprising since it is a natural fact that a decrease of L will enhance sensibility of the soliton shape to dispersive interlayer interactions. As a concluding remark, the new analytical gap-soliton derived above are artificial objects consisting of a bunch

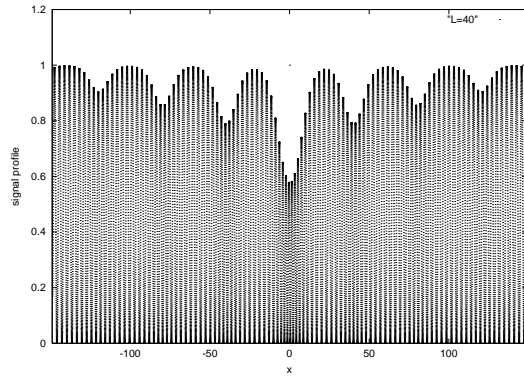


FIG. 5: Interlayer-dispersion-induced localization of the gap-soliton lattice: $L = 40$.

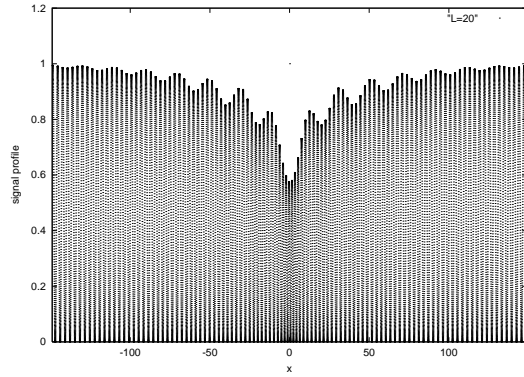


FIG. 6: The gap-soliton lattice for $L = 20$.

of single localized gap-solitons which are compeled to share the same propagation medium, as opposed to the well-known periodons of finite single dielectric layers see e.g. Lidorskis et al. [3]. Therefore, they can be looked on as compressed single gap-solitons where the global mean-field feedback plays role of a compression potential. Since this global feedback forms from fundamental modes of each single dielectric layer, the thickness of its wells and amplitude of the potential barriers will be fixed by the

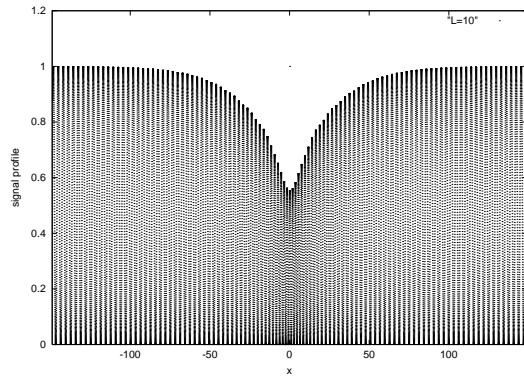


FIG. 7: The gap-soliton lattice for $L = 10$.

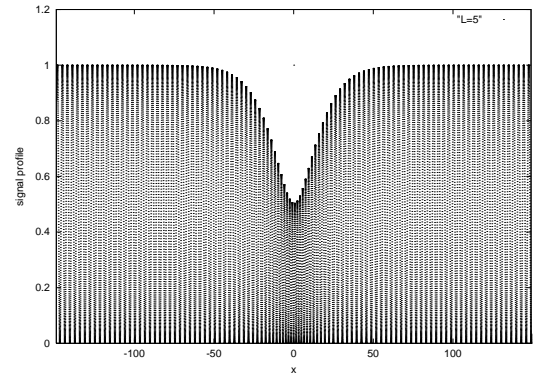


FIG. 8: The gap-soliton lattice for $L = 5$.

single-soliton width and gap, respectively. That is to say, one can monitor the soliton compression within the global feedback. According to our analytical soliton solutions, the waveshape acquired by the gap soliton passing through each dielectric layer reflects features of the compression potential, thus each of these layers actually convey a bunch of compressed single pulses. Within the bunch state the soliton gap is lowered, decreasing with a decrease of the modulus ν . Recall that a decrease of ν brings single solitons closer and closer. As the dispersive inter-layer interaction just sharpens the soliton-lattice on single layers, we also expect similar features (namely a decrease of the soliton gap) to show up in this last context for a relatively large number of interacting single dielectric layers.

Acknowledgments

Part of this work was carried out as A. M. Dikandé was guest at the Abdus Salam International Centre for Theoretical Physics (ASICTP) Trieste, Italy.

-
- [1] W. Chen and D. L. Mills, Phys. Rev. Lett.58, 160(1987).
 - [2] Qiming Li, C. T. Chan, K. M. Ho and C. M. Soukoulis, Phys. Rev. B53, 15577(1996).
 - [3] E. Lidoriskis, K. Busch, Qiming Li, C. T. Chan and C. M. Soukoulis, Phys. Rev. B56, 15090(1997).
 - [4] E. Yablonovitch, Phys. Rev. Lett.58, 2059(1987).
 - [5] S. John, Phys. Rev. Lett.58, 2486(1987).
 - [6] J. D. Joannopoulos, R. D. Meade and J. N. Winn, *Photonic Crystals*(Princeton University Press, Princeton, N.J., 1995).
 - [7] M. Abramowitz and I. Stegun, *Handbook of Mathematical Functions*(Dover, New York , 1964).
 - [8] F. M. Arscott and I. M. Khabaza, *Table of Lamé Polynomials* (Pergamon, London, 1962) vol. 17.
 - [9] B. Sutherland, Phys. Rev. A 8, 2514(1973).
 - [10] A. M. Dikandé, Phys. Scrip. 60, 291(1999).
 - [11] S. Y. Mensah, A. M. Dikandé, F. K. A. Allotey and N. G. Mensah, Superlat. and Microstruct.27, 31(2000).
 - [12] P. G. Kevrekidis, Y. B. Gaididei, A. R. Bishop and A. Saxena, Phys. Rev. E64, 066606(2001).